

Name \_\_\_\_\_ Student Number \_\_\_\_\_

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

- (1) Find
- $y'$
- when
- $y^2 + \cos(x^2) = e^{xy}$

Differentiate implicitly:

$$\begin{aligned}\frac{d}{dx}(y^2 + \cos(x^2)) &= \frac{d}{dx}e^{xy} \\ 2yy' - 2x \sin(x^2) &= (y + xy')e^{xy} \\ y'(2y - xe^{xy}) &= 2x \sin(x^2) + ye^{xy} \\ y' &= \frac{2x \sin(x^2) + ye^{xy}}{2y - xe^{xy}}\end{aligned}$$

- (2) Find
- $f'(x)$
- when
- $f(x) = \sin(|x|)$
- .

Use the chain rule:

$$\begin{aligned}f'(x) &= \frac{d}{dx} \sin(|x|) \\ &= \cos(|x|) \frac{d}{dx} |x| \\ &= \cos(|x|) \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases} \\ &= \begin{cases} \cos(x), & x > 0 \\ -\cos(x), & x < 0 \end{cases}\end{aligned}$$

Note that  $f(x)$  is *not* differentiable at  $x = 0$  since  $|x|$  is not differentiable at  $x = 0$ .

Over  $\rightarrow$

(3) Calculate  $\frac{d}{dx} \left( \sqrt[4]{\tan^{-1}(x^2 - 1)} \right) =$

$$\begin{aligned} & \frac{1}{4} \left( \tan^{-1}(\sqrt{x^2 - 1}) \right)^{-\frac{3}{4}} \frac{d}{dx} \tan^{-1}(x^2 - 1) \\ &= \frac{1}{4} \left( \tan^{-1}(\sqrt{x^2 - 1}) \right) \frac{1}{1 + (x^2 - 1)^2} 2x \end{aligned}$$

Do not simplify any further than this.

(4) Calculate the derivative of

$$y = \frac{(x^2 + 1)(x - \sin(x))^2}{2^x \sqrt{x}}$$

Use logarithmic differentiation:

$$\begin{aligned} \ln y &= \ln \left( \frac{(x^2 + 1)(x - \sin(x))^2}{2^x \sqrt{x}} \right) \\ &= \ln(x^2 + 1) + \ln(x - \sin x)^2 - \ln 2^x - \ln \sqrt{x} \\ &= \ln(x^2 + 1) + 2 \ln(x - \sin x) - x \ln 2 - \frac{1}{2} \ln x \end{aligned}$$

Differentiate both sides implicitly

$$\frac{y'}{y} = \frac{1}{x^2 + 1} 2x + 2 \frac{1}{x - \sin x} (1 - \cos x) - \ln 2 - \frac{1}{2x}$$

If you did this without log. diff. then you did too much work and won't get full marks for the question.